

Abstracts: Dynamics on Lie Groups II

Lodha Mathematical Sciences Institute

7 September to 11 September, 2026

Dynamics on Lie Groups II opens with talks on anti-de Sitter geometry and Mess's theorem (Mazzoli), G-webs and character varieties (Douglas), and the start of Labourie's mini-course on Θ -positive representations of surface groups.

Titles and abstracts

Speaker: Filippo Mazzoli

Title: An introduction to anti-de Sitter geometry

Abstract: The aim of this minicourse is to provide an introduction to anti-de Sitter geometry and, in particular, to describe the classification of maximal globally hyperbolic, Cauchy-compact 3-manifolds. We will start with the definition of anti-de Sitter space, exploring its properties, geodesics, isometries, and various models. We will then focus on the case of dimension 3 and investigate the relationship between anti-de Sitter and 2-dimensional hyperbolic geometry. We will describe a proof of Mess's theorem, showing that these specific spacetimes can be classified and parameterized by the product of two copies of Teichmüller space. If time permits, we will discuss how various phenomena from anti-de Sitter geometry can be generalized to $SO(p, q)$ higher Teichmüller spaces.

Speaker: Daniel Douglas

Title: Webs

Abstract: For an algebraic group G , the point of view of "G-webs" offers a pictorial way to think about the coordinate ring of the algebraic variety of G -characters on a manifold with finitely-generated fundamental group. Webs also appear at the intersection of (quantum) topology and statistical mechanics, in the context of "G-dimer models".

Speaker: François Labourie (*6-hour mini-course)

3-hours in the week of Sept 7- 11: Mon-Tue-Wed 11-12

3-hours in the week of Sept 14-18: Mon (11-12 and 2-3), Tue (11-12)

Title: Θ -positive representations of surface groups

Abstract: Θ -positive representations of surface groups are a certain type of representation of the fundamental group of a closed surface in some semi-simple

groups, which seem to play a significant role amongst representations of surface groups. In this lecture, we will tour the subject and try to get a feeling that these representations form connected components of character varieties. The prototype of these connected components is the Teichmüller space.

- 1) We will first describe and give a dynamical definition of generalised flag manifolds. We will then explain that these manifolds can be listed using an algebraic definition.
- 2) We will then explain what a Θ -positive structure is on a generalised flag manifold again using a dynamical approach, and its consequences in defining positive quadruples, positive curves, positive representations, and give examples.
- 3) We will then move to give a definition of Anosov representations and explain that positive representations are a special case of Anosov representations. We will then show that positive representations form an open set in the set of Anosov representations and give a criterion (*) showing that a limit of positive representations is itself positive.
- 4) We finally show, thanks to a collar lemma akin to the celebrated collar lemma in hyperbolic geometry, that criterion (*) is actually always satisfied. We will explain the idea behind the proof of the collar lemma.

These lectures are based on the work of Guichard—Wienhard, Guichard—Labourie—Wienhard, Beyrer—Guichard—Labourie—Pozzetti-Wienhard. This will be a Higgs bundle-free series of lectures, and I plan to adapt its level to the culture of the audience. Item (3) might be contracted.