

Seminar talk:

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Triality with Schemes and Without Octonions

Triality refers to the unique phenomenon that certain linear algebraic groups of type  $D_4$  have an order three outer automorphism. Outer automorphisms of a linear algebraic group are constrained by automorphisms of the group's Dynkin diagram, and all Dynkin diagrams besides  $D_4$  have either no symmetries or only a single reflection. Classically, working over fields, triality was studied through the use of Octonion algebras and then later through a related structure called a symmetric composition algebra. Octonion algebras by definition are 8 dimensional, and their existence may be considered philosophically equivalent to the existence of triality. These algebras were used to show how the order three automorphism arises from a decomposition of the Clifford algebra, and then this special decomposition can be twisted to produce many other triality related facts such as cohomological identities. In this talk we will begin by reviewing this classical approach to triality, however we will then provide an alternative to using objects which only exist in dimension 8. We highlight how the phenomena of triality can be identified by using concrete Chevalley generators for groups of type  $D$  to study how a decomposition of the Clifford algebra which exists in all dimensions happens to behave uniquely in dimension 8. Ultimately, one may choose the approach they prefer since both approaches generalize to hold over a general scheme and allow us to describe triality in this context. This allows us to provide a description of the category of simply-connected (or adjoint) groups of type  $D_4$  over a scheme in terms of trialitarian algebras, akin to the descriptions for other types in terms of algebras with involution.

Schedule: February 19, 2:30-3:30 p.m.