

Minicourse

Title : Swinnerton-Dyer's method with a view towards the arithmetic of del Pezzo surfaces of degree 4

Speaker : Julian Lyczak (online)

Summary : In this minicourse we will discuss the arithmetic properties of del Pezzo surfaces of degree 4. The leading conjecture by Colliot-Thélène and Sansuc states that the Brauer-Manin obstruction is the only one to both weak approximation and the Hasse principle on such varieties. We will touch upon many techniques and results from the literature to prove this conjecture in subfamilies, but the main focus will be the applications of Swinnerton-Dyer's method for the study of fibrations of twists under abelian varieties.

Colliot-Thélène, Skorobogatov and Swinnerton-Dyer used this method to prove the Hasse principle and weak approximation on some quartic del Pezzos which admit a fibration into genus 1 curves, which was then later extended by Wittenberg. Both results are conditional on Schinzel's conjecture, and the conjectured finiteness of relevant Tate-Shafarevich groups.

A more recent development is the study of quartic del Pezzo surfaces through Kummer surfaces and families of abelian surfaces. Morgan and Skorobogatov used this to remove the dependency of some of Wittenberg's results on Schinzel's hypothesis.

All results above only apply to quartic del Pezzo surfaces with a Brauer group modulo constants of order at most 2. Lastly, we will discuss recent work together with Morgan in which the arithmetic of an infinite family of quartic del Pezzo surfaces with Brauer group of order 4 is successfully studied.

On the way we will encounter results from descent on abelian varieties, in particular Selmer groups in (quadratic) families and the Cassels-Tate pairing. From the descent method we will see results conditional on Schinzel's hypothesis, and unconditional results in certain geometric situations and for zero-cycles.

Schedule :

Tuesday 21st April 2pm to 4pm

Wednesday 22nd April 2pm to 4pm