Workshop: Problems and Techniques in Arithmetic Statistics

Monday

Session 1

- Speaker: Manjul Bhargava
- Title: Indices of Groups, Malle's Conjecture, and Random Integer Polynomials
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 2

- Speaker: Takashi Taniguchi
- Title: Exponential sums for prehomogeneous and coregular spaces and arithmetic statistics
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 3

- Speaker: Fabian Gundlach
- Title: Asymptotics of extensions of simple Q-algebras
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: It is a famous open question in arithmetic statistics how many degree n extensions L of a given number field K there are with bounded discriminant (asymptotically, as the discriminant bound goes to infinity). We will discuss joint work with Béranger Seguin on a non-commutative variant of this problem. Any extension L of a division Q-algebra K can be uniquely decomposed into an inner Galois extension L/E and an outer extension E/K. We explain how to count inner Galois extensions using Brauer groups, and why counting outer extensions is essentially equivalent to the commutative question.

Session 4

- Speaker: Tom Fisher

- Title: Visible 3-torsion in the Tate-Shafarevich group of a genus 2 Jacobian
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: I will describe joint work in progress with Lazar Radicevic where we compute explicit equations for 3-coverings of genus 2 Jacobians that violate the Hasse principle. Since this is a situation where classical descent methods are not practical, we instead use the method of visibility introduced by Cremona and Mazur. The 3-coverings are written as the singular locus of a cubic form in 9 variables, and can also (when the genus 2 curve has a rational Weierstrass point) be conveniently described by a trivector.

Tuesday

Session 1

- Speaker: Sameera Vemulapalli
- Title: Brill-Noether theory of smooth plane curves
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: Given a smooth curve C, it is natural to ask: what are all the degree d maps from C into a projective space Pr? The study of this question is called Brill-Noether theory. In this talk, I'll describe joint work with Hannah Larson, in which we describe the Brill-Noether theory of general smooth plane curves. Usually, results in this area are proved using deformation theory. By contrast, in this case the main tool is Wood's parametrization of ideal classes of rings associated to binary n-ic forms, which has proved immensely useful in arithmetic statistics. In particular, we see that the parametrizations of arithmetic statistics can also be useful to address classical questions in algebraic geometry.

Session 2

- Speaker: Gaurav Patil

- Title: Weakly Divisible Rings and Lower Bounds on S_n - Number Fields with fixed degree and bounded discriminant
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 3

- Speaker: Asvin G
- Title: Stacky resolutions of the multi-discriminant locus and a conjecture on splitting densities
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: We construct a new resolution of singularities (using Artin stacks) of the locus of tuples of polynomials sharing some roots. Using this very explicit resolution, we resolve a conjecture on the splitting densities of polynomials over a p-adic field formulated by Bhargava, Cremoma, Fisher and Gajović. This conjecture points the way towards some deeper symmetries that we might indicate if time permits.

Session 4

- Speaker: Ronno Das
- Title: Rational curves on blowups
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: Inspired by Segal's work on rational functions and its many extensions, we show that the space of holomorphic maps from the Riemann sphere to certain blowups of projective space has homological stability as the degree gets large. Via the Grothendieck-Lefschetz trace formula and Katz--Milnor bounds on Betti numbers, we obtain a version of the Manin conjecture for degree 4 del Pezzo surfaces over finite fields. The talk will be based on joint work with Brian Lehmann, Sho Tanimoto and Philip Tosteson.

Session 5

- Speaker: Aaron Landesman
- Title: Malle's conjecture over function fields
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: For G a finite group, Malle's conjecture predicts the asymptotic growth of the number of G extensions of a fixed global field. In joint work with Ishan Levy, we compute the asymptotic growth of the number of Galois G extensions of Fq(t), for q sufficiently large and relatively prime to |G|.

Thursday

Session 1

- Speaker: Brandon Alberts
- Title: A Crash Course in the Selberg-Delange Method
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 2

- Speaker: Ari Shnidman
- Title: Hecke reciprocity and class groups
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: Hecke's theorem states that the class of the difference of F/Q is a square in the class group. I'll explain how this affects the (conjectural) distribution of the 2-parts of class groups in certain special families of number fields, such as the family of Kummer extensions F_n = Q(n^{1/p}). When p = 3, we prove aspects of this Cohen-Lenstra type heuristic, by showing that the average size of Cl_{F_n}[2] is equal to 2 (resp 3/2) depending on whether F_n is wildly (resp tamely) ramified at p. The key technical ingredient is our

previous work with Alpoge and Bhargava on counting integral G(Q)-orbits in a certain G-invariant quadric. Joint work with Artane Siad.

Session 3

- Speaker: Shiva Chidambaram
- Title: Cohen-Lenstra type conjectures for p-torsion in characteristic p
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 4

- Speaker: Tim Santens
- Title: Ray class groups and Brauer-Manin obstructions
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: I will describe and give examples of certain surprising features of ray class groups of number fields which can be explained via Brauer-Manin obstructions on certain stacks. Based on these features and work of Loughran and I on the leading constant in Malle's conjecture I will also state some conjectures on the statistical properties of ray class groups. This is joint work with Daniel Loughran and Ross Paterson.

Session 5

- Speaker: Kiran Kedlaya
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Friday

Session 1

- Speaker: Alina Bucur
- Title: Number field counting via multiple Dirichlet series
- Location: Harish Chandra Lecture Hall, LMSI, NCP

- Abstract: We will discuss how to use multiple Dirichlet series to set up a generating function that helps us count number fields of fixed degree and Galois group by general invariants. The strategy works for certain semiconcentrated Galois groups, but it is limited by (sub)convexity bounds for the growth of L-functions inside the critical strip. This is joint work with Brandon Alberts.

Session 2

- Speaker: Alison Miller
- Title: Asymptotics for Sp_{2n}-orbits on quadratic forms
- Location: Harish Chandra Lecture Hall, LMSI, NCP
- Abstract: This talk will center on an asymptotic formula for an (appropriately weighted) count of integer orbits of the symplectic group on symplectic matrices, ordered by invariant height. This formula can be seen as an analogue of formulas going back to Gauss for orbits of binary quadratic forms, and also has analogues for adjoint representations of more general semisimple algebraic groups. We'll discuss applications to counting other objects, such as relative ideal classes in quadratic extensions of monogenic fields and Seifert matrices from knot theory.

Session 3

- Speaker: Dr. Zahra Janbazi
- Title: Finiteness Theorems in Arithmetic Statistics
- Location: Harish Chandra Lecture Hall, LMSI, NCP

Session 4

- Speaker: Anders Sodergren
- Title: Counting cubic fields and low-lying zeros of Artin L-functions
- Location: Harish Chandra Lecture Hall, LMSI, NCP

- Abstract: In this talk I will describe the relation between counting results for cubic fields with prescribed splitting conditions and the low-lying zeros of certain Artin L-functions. The main application of this investigation is a conditional omega result for cubic field counting functions. This is joint work with Peter Cho, Daniel Fiorilli and Yoonbok Lee. If time allows, then I will also discuss Victor Ahlquist's recent work that generalizes the above to the function field setting.